Sinefitting : Robust Curvature Estimator On Surface Triangulation

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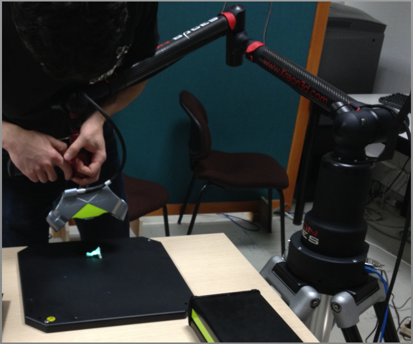
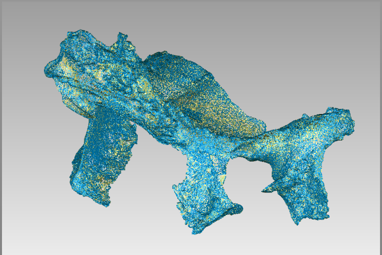
To obtain a surface variation descriptor on unstructured data.


[1]

Point Cloud



[1] Acquilon scanner of Kreon

Curvature estimation methods generally divided into two parts:

- Normal estimation
- Curvature tensor estimation itself

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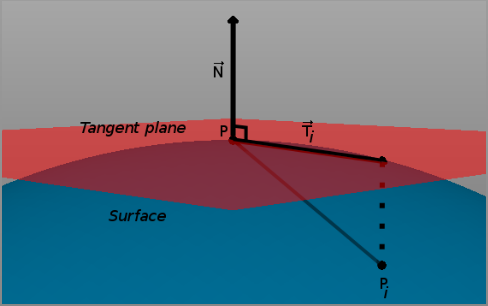
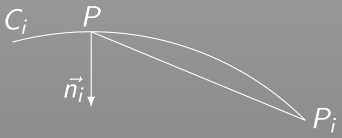
For the evaluation of the curvature estimators we use 3 criterions :

- Pointwise Convergence
- Precision
- Robustness

Notations 1/2 (Neighborhood & plane section)

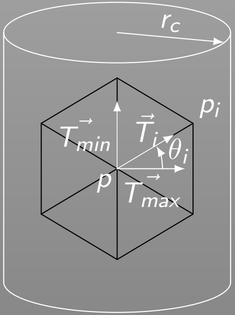




C_i is the normal section containing P_i

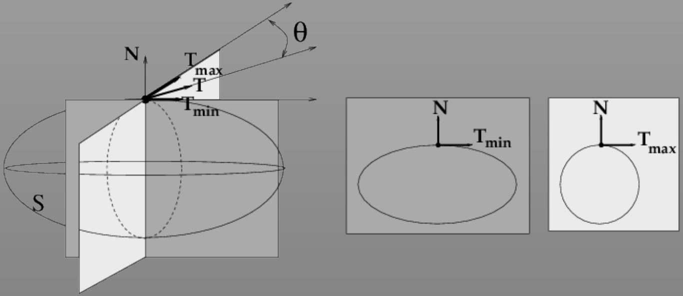
- P: target
- P_i: neighbors
- $\vec{N}$: normal of the surface at P
- $\vec{T}_i$: tangent of C_i at P
- $\vec{n}_i$: normal of C_i at P
- k_i: curvature of C_i at P

Notations 2/2 (Principal directions & curvatures)



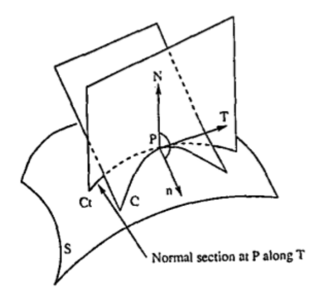
- k_{max}&k_{min}: maximal and mininal curvatures
- K_H: Mean curvature: $K_H = (k_{max} + k_{min})/2$
- K_G: Gaussian curvature: $K_G = k_{max} * k_{min}$
- $\vec{T}_{max}$&$\vec{T}_{min}$: respectively k_{max}&k_{min} directions
- θ_i: angle between $\vec{T}_{max}$ and $\vec{T}_i$

Euler theorem



$$k_i = k_{max} cos^2(\theta_i) + k_{min} sin^2(\theta_i) \quad (1)$$

Meusnier theorem



2

$$k_i = k.cos(\beta) \tag{2}$$

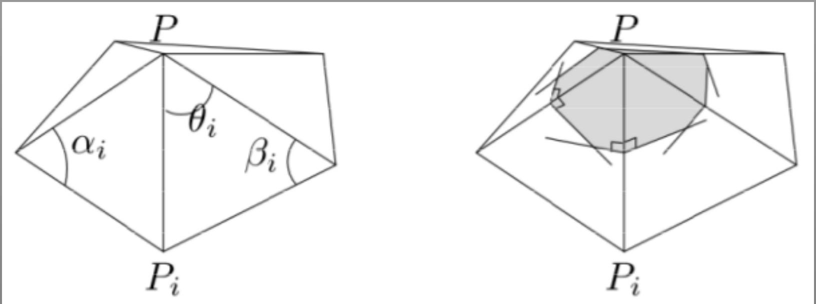
Where β is the angle between $\vec{n}$ and $\vec{N}$
and k is the curvature of C at the point P

[2]Illustration extracted from Chen and Schmitt book [CS92]

We can classify curvature estimators in three classes:

- Averaging methods (Meyer's et al. method [MMB02] (`SDA`))
- Surface fitting methods (Mc Ivor's et al. method [MW97] (`SQFA`))
- Curve fitting methods (Chen's, Taubin's and Langer's methods)

Meyer et al. [MMB02] (SDA)



[3]

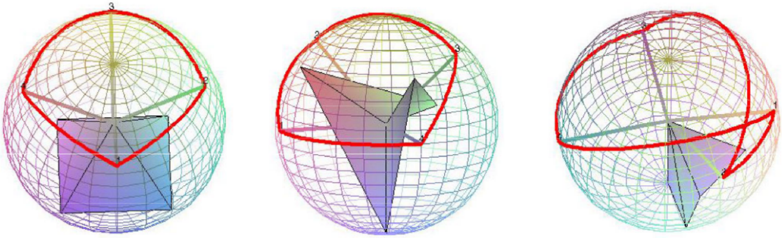
Angle weighted area or Voronoï area around vertex P in grey

This method just computes K and H

[3] Illustration extracted from Bac et al. [BDM05]

Meyer et al. [MMB02] (SDA)

Weakness of this method:



Convex, saddle and fan vertices and their Gaussian indicatrix [3]

[3] Illustration extracted from Bac et al. [BDM05]

Mc Ivor et al. [MW97]: Simple Quadratic Fitting (SQFA)

Consists in solving an equation like eq.(3) by using the spatial coordinates of each P_i

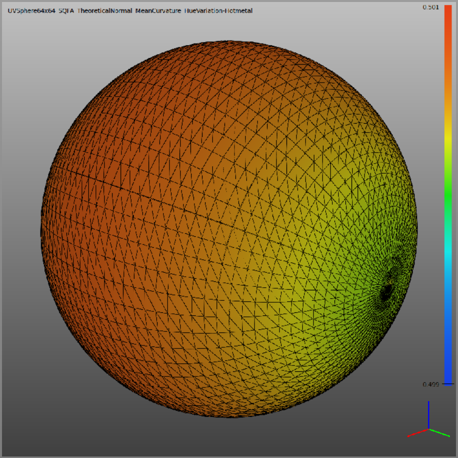
$$z = ax^2 + by^2 + cxy \tag{3}$$

Is an overdetermined system usually solved by least squares.

Researched values are obtained by using the coefficients.

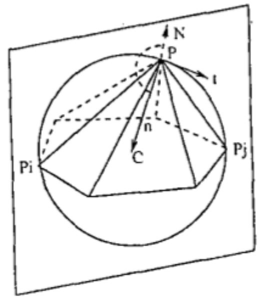
Mc Ivor et al. [MW97]: Simple Quadratic Fitting (SQFA)

Weakness of this method:



Highly sensitive to the distrubution of the neighborhood.

Chen & Smith [CS92]



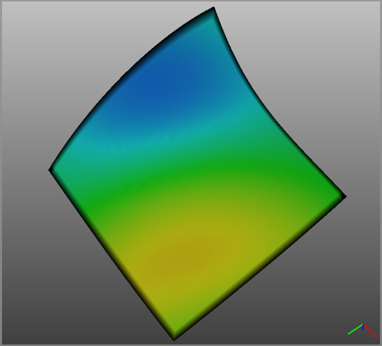
[4]

1. Find the most opposite triplets
2. Compute k for each circle fitted over each choosen triplet
3. Use the Meusnier theorem to evaluate the k_i
4. Finally, fit a transformed equation of the Euler theorem.

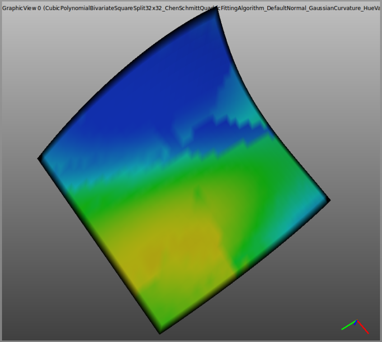
[4] Illustration extracted from Chen and Schmitt book [CS92]

Chen & Smith [CS92]

Weakness of this method:



Theoretical curvature



Chen K_G estimation

Local instabilities at saddle point and low curvature.

Taubin [TFA95] and Langer [LBS07]

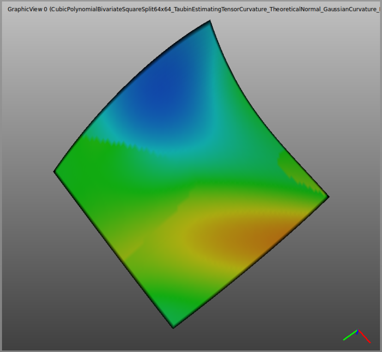
Firstly: both methods compute k_i as $k_i \approx \frac{2\vec{N}^t(\vec{PP_i})}{||\vec{PP_i}||^2}$

Secondly:

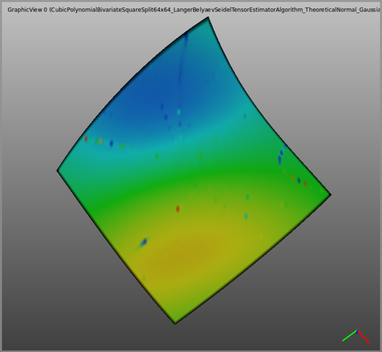
- Taubin gives a matricial system representation of the curvature tensor.
- Whereas Langer evaluate the curvature as two integrals modeling K_H and K_G.

Taubin [TFA95] and Langer [LBS07]

Weakness of this methods:



Taubin K_G estimation



Langer K_G estimation

Taubin is imprecise and Langer has occasional errors

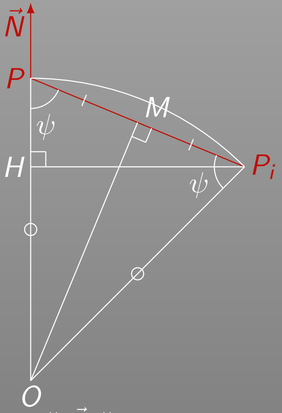
All this curvature estimators present dysfunctions

Can we find a new curvature estimator less sensitive to neighborhood geometry ?

The `SineFitting` algorithm is composed of two steps

1. Evaluation of k_i as in Taubin and Langer algorithms by circle fitting.
2. Fitting a transformed equation of Euler theorem as in Chen algorithm but without using Meunsier theorem.

(Recall Euler equation) $k_i = k_{max} cos^2(\theta_i) + k_{min} sin^2(\theta_i)$



Known data

$$\cos\psi = \frac{||\vec{PH}||}{||\vec{PP_i}||} = \frac{||\vec{MP_i}||}{||\vec{OP_i}||};\ \ldots;\ ||\vec{OP_i}|| = r_i = |\frac{1}{2}.\frac{||\vec{PP_i}||^2}{\vec{PP_i}.\vec{N}}|;\ k_i = 2.\frac{\vec{PP_i}.\vec{N}}{||\vec{PP_i}||^2}$$

$\vec{T_{max}}$ is unknown, so θ_i cannot be directly computed.
Let φ an angle such that $\theta_i = \alpha_i + \varphi$, where $\alpha_i = \angle(\vec{T}_0, \vec{T}_i)$

Euler equation is rewritten as:

$$k_i = k_{max} \cos^2(\alpha_i + \varphi) + k_{min} \sin^2(\alpha_i + \varphi)$$

...

$\vec{T}_{max}$ is unknown, so θ_i cannot be directly computed.
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Euler equation is rewritten as:

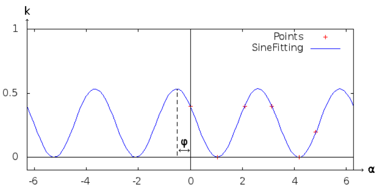
$$k_i = k_{max}\cos^2(\alpha_i + \varphi) + k_{min}\sin^2(\alpha_i + \varphi)$$

$$\ldots$$

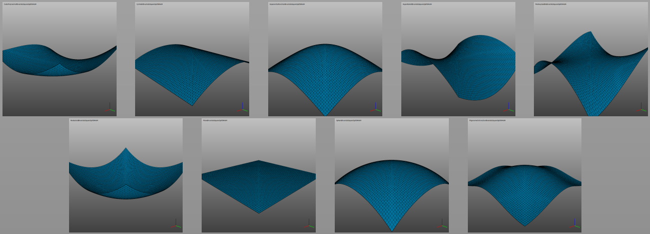
$$k_i = a\cos(2\alpha_i) + b\sin(2\alpha_i) + c$$

where (if $a > 0$ for example),

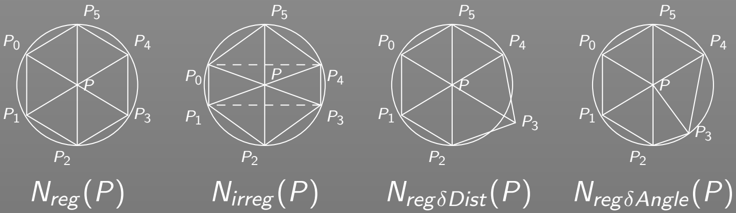
$$\varphi = -\frac{\tan^{-1}(\frac{b}{a})}{2}, k_{max} = c + \sqrt{a^2 + b^2}, k_{min} = c - \sqrt{a^2 + b^2}$$



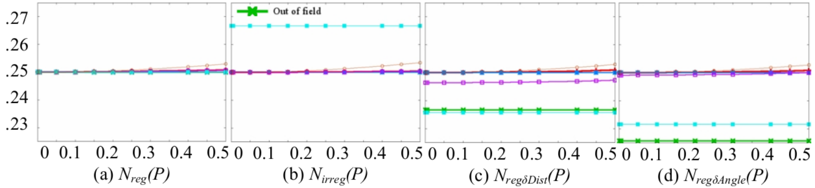
Hamann's discretization surface for robustness [Ham91]



Different convergent discretisation methods of mathematical surfaces. Called `NeighborDealers`

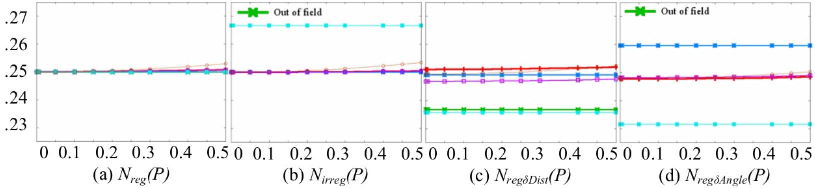


Pointwise convergence



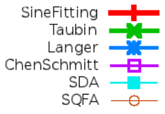
(a) $N_{reg}(P)$ (b) $N_{irreg}(P)$ (c) $N_{reg\delta Dist}(P)$ (d) $N_{reg\delta Angle}(P)$

Cylinder pointwise convergence test for mean curvature computation with theoretical normal.

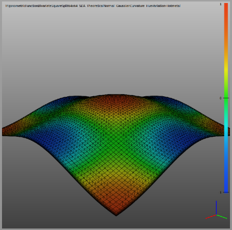


(a) $N_{reg}(P)$ (b) $N_{irreg}(P)$ (c) $N_{reg\delta Dist}(P)$ (d) $N_{reg\delta Angle}(P)$

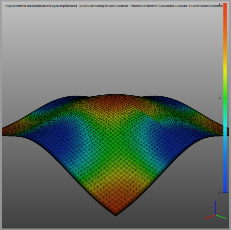
Cylinder pointwise convergence test for mean curvature computation with area-weighted normal.



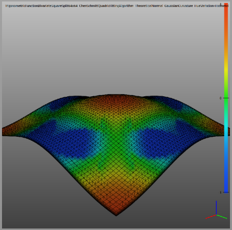
Precision



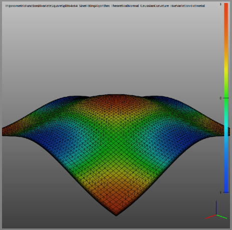
SDA



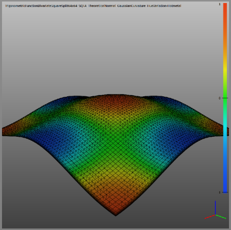
Taubin



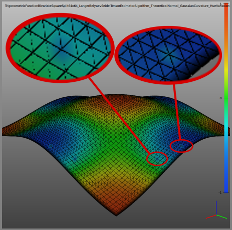
Chen



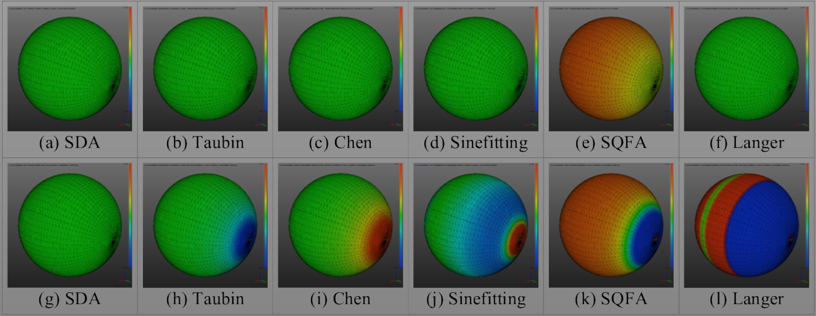
SineFitting



SQFA



Langer



(a) SDA (b) Taubin (c) Chen (d) Sinefitting (e) SQFA (f) Langer

(g) SDA (h) Taubin (i) Chen (j) Sinefitting (k) SQFA (l) Langer

Mean curvature evaluation based on theoretical and area-weighted normal

Conclusion:

- According to the performed tests, `Sinefitting` is not always the most accurate method, but is far more stable.
- It is easy to implement.

Perspectives:

- Test robustness on noised data following perturbations of Gatzke [GG06].
- Experiment on point cloud.

Future work:

- We will try to use the same intuition for the normal estimator.

Thank you for listening

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Experiment platform: http://smithdr.labri.fr/

All results are available on:

http://dept-info.labri.fr/~ charton/curvature_analysis/

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