Algorithm 1 SineFitting Proof

$k_i = k_1 \cos^2(\alpha_i + \varphi) + k_2 \sin^2(\alpha_i + \varphi)$
$k_i = k_1 \cos^2(\alpha_i + \varphi) + k_2 - k_2 \cos^2(\alpha_i + \varphi)$
$k_i = (k_1 - k_2) \cos^2(\alpha_i + \varphi) + k_2$

`Let` $a \in \mathbb{R}^{*+}, b \in \mathbb{R}, c \in \mathbb{R} /\ \varphi = -\frac{\tan^{-1}(\frac{b}{a})}{2}, k1 = c + \sqrt{a^2 + b^2}, k2 = c - \sqrt{a^2 + b^2}$
$k_i = ((c + \sqrt{a^2 + b^2}) - (c - \sqrt{a^2 + b^2})) \times \cos^2(\alpha_i - \frac{\tan^{-1}(\frac{b}{a})}{2}) - \sqrt{a^2 + b^2} + c$
$k_i = 2\sqrt{a^2 + b^2} \times \cos^2(\alpha_i - \frac{\tan^{-1}(\frac{b}{a})}{2}) - \sqrt{a^2 + b^2} + c$
$k_i = 2\sqrt{a^2 + b^2} \times (\frac{1}{2}(\cos(2\alpha_i - \tan^{-1}(\frac{b}{a})) + 1)) - \sqrt{a^2 + b^2} + c$
$k_i = \sqrt{a^2 + b^2} \times \cos(2\alpha_i - \tan^{-1}(\frac{b}{a})) + c$
$k_i = \sqrt{a^2 + b^2} \times [\cos(2\alpha_i) \cos(\tan^{-1}(\frac{b}{a})) + \sin(2\alpha_i) \sin(\tan^{-1}(\frac{b}{a}))] + c$
$k_i = \sqrt{a^2 + b^2} \times [\frac{\cos(2\alpha_i)}{\sqrt{1+\frac{b^2}{a^2}}} + \sin(2\alpha_i) \frac{b}{a} \frac{1}{\sqrt{1+\frac{b^2}{a^2}}}] + c$
$k_i = \sqrt{a^2 + b^2} \times [\frac{a \cos(2\alpha_i) + b \sin(2\alpha_i)}{a\sqrt{1+\frac{b^2}{a^2}}}] + c$
$k_i = \sqrt{a^2 + b^2} \times [\frac{a \cos(2\alpha_i) + b \sin(2\alpha_i)}{\sqrt{a^2+b^2}}] + c$
$k_i = a \cos(2\alpha_i) + b \sin(2\alpha_i) + c$

`In case` $a \in \mathbb{R}^{*-}$ `Let` $d \in \mathbb{R}^{*+} / d = -a$
$\varphi = -\frac{\pi - \tan^{-1}(\frac{b}{d})}{2}, k1 = c + \sqrt{d^2 + b^2}, k2 = c - \sqrt{d^2 + b^2}$
$k_i = ((c + \sqrt{d^2 + b^2}) - (c - \sqrt{d^2 + b^2})) \times \cos^2(\alpha_i - \frac{\pi - \tan^{-1}(\frac{b}{d})}{2}) - \sqrt{d^2 + b^2} + c$
$k_i = 2\sqrt{d^2 + b^2} \times \cos^2(\alpha_i - \frac{\pi - \tan^{-1}(\frac{b}{d})}{2}) - \sqrt{d^2 + b^2} + c$
$k_i = 2\sqrt{d^2 + b^2} \times (\frac{1}{2}(\cos(2\alpha_i - (\pi - \tan^{-1}(\frac{b}{d}))) + 1)) - \sqrt{d^2 + b^2} + c$

$k_i = \sqrt{d^2 + b^2} \times -\cos(2\alpha_i + \tan^{-1}(\frac{b}{d}))) + c$
$k_i = \sqrt{d^2 + b^2} \times -[\cos(2\alpha_i) \cos(\tan^{-1}(\frac{b}{d})) - \sin(2\alpha_i) \sin(\tan^{-1}(\frac{b}{d}))] + c$
$k_i = \sqrt{d^2 + b^2} \times [-\frac{\cos(2\alpha_i)}{\sqrt{1+\frac{b^2}{d^2}}} + \sin(2\alpha_i) \frac{b}{d} \frac{1}{\sqrt{1+\frac{b^2}{a^2}}}] + c$
$k_i = \sqrt{d^2 + b^2} \times [\frac{-d \cos(2\alpha_i) + b \sin(2\alpha_i)}{d\sqrt{1+\frac{b^2}{a^2}}}] + c$
$k_i = \sqrt{d^2 + b^2} \times [\frac{-d \cos(2\alpha_i) + b \sin(2\alpha_i)}{\sqrt{d^2+b^2}}] + c$
$k_i = -d \cos(2\alpha_i) + b \sin(2\alpha_i) + c$
$k_i = a \cos(2\alpha_i) + b \sin(2\alpha_i) + c$

`In case` $a = 0$
$k_i = b \sin(2\alpha_i) + c$
$k_i = b \cos(2\alpha_i - \pi/2) + c$
$k_i = b \cos(2(\alpha_i - \pi/4)) + c$
$k_i = b(\cos^2(\alpha_i - \pi/4) - \sin^2(\alpha_i - \pi/4)) + c$
$k_i = b \cos^2(\alpha_i - \pi/4) - b \sin^2(\alpha_i - \pi/4) + c$
$k_i = b \cos^2(\alpha_i - \pi/4) - b + b \cos^2(\alpha_i - \pi/4) + c$
$k_i = 2b \cos^2(\alpha_i - \pi/4) - b + c$
$\varphi = -sign(b) * \pi/4, k1 = -b + c + |b|, k2 = -b + c - |b|$